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Remarks on the rolling tachyon BCFT

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Abstract: It is shown how the boundary correlators of the euclidean theory corresponding to the rolling tachyon solution can be calculated directly from Sen's boundary state. The resulting formulae reproduce precisely the expected perturbative open string answer. We also determine the open string spectrum and comment on the implications of our results for the timelike theory.

Keywords: Bosonic Strings, D-branes, Tachyon Condensation.

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1. Introduction

The study of time dependent string theory solutions has recently been a very active area of research. The creation and decay of an unstable D-brane constitutes one of the simplest examples of a time dependent background. Sen [1, 2] showed that the rolling of the open string tachyon in time can be described by the boundary deformation

$$S = -\frac{1}{2\pi}\int_{\Sigma} d\tau d\sigma\, \partial_a X^0 \partial^a X^0 + \lambda \int_{\partial\Sigma} d\tau \cosh\left(\sqrt{2}X^0(\tau)\right). \tag{1.1}$$

The boundary deformation in (1.1) can be interpreted as a non-zero background for the tachyon field $T(X^0) = \lambda\cosh(\sqrt{2}X^0)$. At early and late times, $X^0 \to \pm\infty$, the tachyon approaches its minimum $T \to \infty$ where the unstable brane has disappeared. Hence (1.1) gives a concrete realisation of a 'S(pacelike)-brane' [3].

Sen showed that (1.1) defines an exact boundary conformal field theory (BCFT) by relating the timelike theory to a spacelike theory by means of the analytic continuation $X^0 \mapsto iX$. The euclidean boundary theory

$$S = \frac{1}{2\pi}\int_{\Sigma} d\tau d\sigma\, \partial_a X \partial^a X + \lambda \int_{\partial\Sigma} d\tau \cos\left(\sqrt{2}X(\tau)\right) \tag{1.2}$$

is then conformal since the boundary perturbation is exactly marginal, as follows from the underlying SU(2) symmetry of a boson compactified on a circle at the self dual radius [4, 5]. In conformal field theory D-branes are described by boundary states; for a non-compact boson associated with the deformation (1.2) the relevant boundary state is [4] and [6]–[9]

$$\|B\rangle\rangle = \frac{1}{2^{1/4}}\sum_{j,m} D^j_{-m,m}\begin{pmatrix}\cos\pi\lambda & -\sin\pi\lambda \\ \sin\pi\lambda & \cos\pi\lambda\end{pmatrix} |j,m,m\rangle\rangle\,, \tag{1.3}$$

where $D^j_{-m,m}(g)$ is the $(-m,m)$-matrix element of $g \in \mathrm{SU}(2)$ in the representation with spin j (which in our conventions is a non-negative half-integer), and $|j,m,m\rangle\rangle$ is the Virasoro Ishibashi state (labelled by j) in the sector with $(p_L,p_R) = 2\sqrt{2}(m,m)$. (Note that m is here also half-integer. An explicit formula for the matrix elements $D^j_{m,n}(g)$ can for example be found in [8].)

A different rolling tachyon, called the 'half S-brane', was introduced in [10, 11]

$$S = -\frac{1}{2\pi}\int_\Sigma d\tau d\sigma\, \partial_a X^0 \partial^a X^0 + \lambda \int_{\partial\Sigma} d\tau e^{\sqrt{2}X^0} . \tag{1.4}$$

This tachyon background describes the decay of an unstable brane since the tachyon approaches its maximum (where the tachyon has not condensed and the D-brane is present) at early times, while it obtains its minimum at late times. An analytic continuation to the spacelike theory produces now

$$S = \frac{1}{2\pi}\int_\Sigma d\tau d\sigma\, \partial_a X \partial^a X + \lambda \int_{\partial\Sigma} d\tau e^{i\sqrt{2}X} . \tag{1.5}$$

The boundary state associated to (1.5) is given by

$$\|B\rangle\rangle = \sum_{j=0,\frac{1}{2},\cdots} \sum_{m\geq 0}^{j} \binom{j+m}{2m} (-\lambda)^{2m} |j,m,m\rangle\rangle . \tag{1.6}$$

This is the boundary state (1.3) corresponding to the group element $g_\lambda \in \mathrm{SL}(2,\mathbb{C})$ [8, 9], where

$$g_\lambda = \begin{pmatrix} 1 & -\lambda \\ 0 & 1 \end{pmatrix} . \tag{1.7}$$

The exact conformal field theory description of the rolling tachyon in terms of its boundary state has been used to investigate many aspects of tachyon condensation and brane decay, in particular, the production of closed strings in the decay [12, 13], tachyon matter [1], two dimensional string theory [14, 15] and comparison to minisuperspace calculations [16, 17].

On the other hand, various aspects of this boundary conformal field theory still remain mysterious. The analytic continuation from the spacelike theory to the timelike theory is still not satisfactorily understood, since it is not clear at which stage in the calculation the analytic continuation should be performed. The timelike boundary states contain components which diverge in the $X^0 \to \infty$ limit [18]. Although the divergent components do not contribute in some calculations (like the bulk one point functions responsible for the closed string creation), they nevertheless play an important role for conserved charges associated with branes [19, 20] and the open string interpretation of the boundary conformal field theory [21].

At an even simpler level, various string amplitudes (of the euclidean or the timelike theory) have only been calculated rather indirectly, in particular, by using the relation of the theory to a particular limit of Liouville theory (see for example [16, 10] and [22]–[25]). However, given the relative simplicity of this model, it should be possible to derive these amplitudes directly from the conformal field theory description of the D-brane in terms of its boundary state. In this paper we want to explain how this can be done, at least for the case of the boundary correlators of the euclidean theory. More precisely, we shall explain how these correlation functions can be obtained directly from the euclidean boundary state (1.6), following ideas of [26]. The resulting expressions reproduce precisely the expected open string expansion[1] that was, for example, used as the starting point in [27]. We also explain how the techniques we describe should allow one to determine other string amplitudes directly from the boundary state.

In this paper we focus on the case of a single free boson that can be described in terms of SU(2) boundary states. The calculational techniques we describe can however also be applied to boundary states that are associated to groups other than SU(2).

The paper is organised as follows. In section 2 we introduce our conventions and explain the calculation of the boundary n-point function of the euclidean theory in detail. We also mention how the open string spectrum can be derived from this boundary state, and how these results fit together. Finally we comment in section 3 on the continuation to the timelike case. We have included an appendix in which some of the more technical parts of our calculations are explained.

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2. Boundary correlators and the cylinder

In this section we want to calculate the boundary correlators of the euclidean rolling tachyon solution at $c = 1$. We begin by describing our conventions in detail.

2.1 Conventions

The spacelike closed string field $X(\sigma, \tau)$ has the mode expansion (in the following we shall always set $\alpha' = 1/2$)

$$X(\sigma,\tau) = x_0 + \frac{1}{2} p\tau + \frac{i}{2} \sum_{n\neq 0} \frac{1}{n} e^{-in\tau} \left(\alpha_n e^{-in\sigma} + \tilde{\alpha}_n e^{in\sigma}\right) . \tag{2.1}$$

The canonical commutation relations are

$$[\alpha_m, \alpha_n] = [\tilde{\alpha}_m, \tilde{\alpha}_n] = m\, \delta_{m,-n} \, , \qquad [x, p] = i \, . \tag{2.2}$$

As usual, the boundary state will be inserted at $\tau = 0$; for the following it is therefore useful to define the positive and negative parts of X at $\tau = 0$:

$$\begin{aligned} X_>(\sigma) &= \frac{i}{2} \sum_{n>0} \frac{1}{n} \left(\alpha_n e^{-in\sigma} + \tilde{\alpha}_n e^{in\sigma}\right) , \\ X_<(\sigma) &= \frac{i}{2} \sum_{n<0} \frac{1}{n} \left(\alpha_n e^{-in\sigma} + \tilde{\alpha}_n e^{in\sigma}\right) . \end{aligned} \tag{2.3}$$

[1] While this is certainly the expected answer, it is unclear to us how to derive it more abstractly since the 'marginal field' by which the boundary state is perturbed is not actually present in the theory at infinite radius. In any case, the detailed relation between these two calculations is instructive, and may be useful in other contexts. For example, our analysis gives rise to a prescription for how to evaluate these formally divergent integrals.

The commutator of the positive and negative modes is

$$
\begin{aligned}
[X_>(\sigma_1), X_<(\sigma_2)] &= \frac{1}{4}\sum_{n>0}\frac{1}{n}\left(e^{-in(\sigma_1-\sigma_2)} + e^{in(\sigma_1-\sigma_2)}\right) \\
&= -\frac{1}{4}\log\left(1-e^{-i(\sigma_1-\sigma_2)}\right) - \frac{1}{4}\log\left(1-e^{i(\sigma_1-\sigma_2)}\right) \\
&= -\frac{1}{4}\log\left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]. \qquad (2.4)
\end{aligned}
$$

If the spacelike boson is compactified on a circle of radius R, then the left- and right-moving momenta are quantised as

$$
(p_L, p_R) = \left(\frac{\hat{m}}{R} + 2\hat{n}R, \frac{\hat{m}}{R} - 2\hat{n}R\right), \qquad (2.5)
$$

where $\hat{m}, \hat{n} \in \mathbb{Z}$. The self-dual radius is $R_{sd} = 1/\sqrt{2}$; at this point, the momenta are simply

$$
(p_L, p_R) = \sqrt{2}\,(\hat{m}+\hat{n}, \hat{m}-\hat{n})\,. \qquad (2.6)
$$

At the self-dual point, the symmetry is enhanced from u(1) to su(2); the relevant (left-moving) operators are (up to cocycle factors) given as

$$
J^{\pm}(z) =: \exp\left(\pm i2\sqrt{2}X_L(z)\right):, \qquad J^3(z) = \sqrt{2}i\partial_z X_L(z)\,, \qquad (2.7)
$$

where $z = e^{i(\tau+\sigma)}$, and

$$
X_L(z) = \frac{1}{2}x_0 - \frac{i}{4}p_L\log(z) + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n\, z^{-n}\,. \qquad (2.8)
$$

(Obviously there is a similar extended right-moving symmetry, generated by corresponding formulae in which $X_L(z)$ is replaced by $X_R(\bar{z})$ with $\bar{z} = e^{i(\tau-\sigma)}$.) The normal ordering in (2.7) is the usual chiral normal ordering, where positive modes are moved to the right of negative modes. The corresponding modes satisfy the standard $\mathrm{su}(2)_1$ commutation relations,

$$
\begin{aligned}
[J^3_m, J^{\pm}_n] &= \pm J^{\pm}_{m+n} \\
[J^+_m, J^-_n] &= 2\,J^3_{m+n} + m\,\delta_{m,-n} \\
[J^3_m, J^3_n] &= \frac{1}{2}m\,\delta_{m,-n}\,. \qquad (2.9)
\end{aligned}
$$

For future reference we observe that $J^3_n = \frac{1}{\sqrt{2}}\alpha_n$.

2.2 Disc amplitudes

We are interested in calculating various disk diagrams involving the boundary state corresponding to the group element $g_\lambda \in \mathrm{SL}(2,\mathbb{C})$ (1.7). As is explained in [4] and [6]–[9], we can write this boundary state also as

$$
\|B\rangle\rangle_\infty = P_\infty \exp\left(-\lambda J^+_0\right)\|e\rangle\rangle\,, \qquad (2.10)
$$

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where J_0^+ is the (left-moving) chiral mode, $\|e\rangle\rangle$ denotes the boundary state corresponding to the identity state for the SU(2) theory at the self-dual radius, and P_∞ is the projector onto states for which $p_L = p_R$. We also write

$$\|N\rangle\rangle \equiv \|e\rangle\rangle_\infty \tag{2.11}$$

for the usual Neumann boundary state (in the uncompactified situation).

We are interested in calculating the n-point function of n boundary vertex operators, using the above boundary state description of the D-brane. Adopting the method of [26] to this context, one can calculate this as follows. We define the *Neumann normal ordering* of an open string vertex operator at $\tau = 0$ as

$$: e^{i\omega X(\sigma)} :_N \equiv e^{2i\omega X_<(\sigma)}\, e^{i\omega x_0}\, e^{i\omega(X_>(\sigma)-X_<(\sigma))}\,. \tag{2.12}$$

Actually, as will be shown in the appendix, this normal ordering prescription corresponds precisely to considering the leading term in the bulk-boundary OPE that describes the limit in which a bulk operator approaches the boundary.[2] In particular, this observation therefore determines the normalisation of our boundary vertex operators: the normalisation of the boundary vertex operators is such that the above bulk-boundary OPE coefficient is unity.

The motivation for the normal ordering prescription (2.12) is that the exponent on the right annihilates the Neumann boundary state $\|N\rangle\rangle$,

$$(X_>(\sigma) - X_<(\sigma))\,\|N\rangle\rangle = \frac{i}{2}\sum_{n>0}\frac{1}{n}\left[(\alpha_n + \tilde\alpha_{-n})\,e^{-in\sigma} + (\tilde\alpha_n + \alpha_{-n})\,e^{in\sigma}\right]\|N\rangle\rangle = 0\,. \tag{2.13}$$

In particular, this implies that the one-point function

$$\langle 0|\ : e^{i\omega X(\sigma)} :_N\ \|N\rangle\rangle = \langle 0|\, e^{i\omega x_0}\,\|N\rangle\rangle \tag{2.14}$$

vanishes unless $\omega = 0$. Furthermore, the higher point functions

$$\langle 0|\ : e^{i\omega_1 X(\sigma_1)} :_N\ \cdots\ : e^{i\omega_n X(\sigma_n)} :_N\ \|N\rangle\rangle \tag{2.15}$$

converge, and agree in fact with the boundary correlators for the corresponding operators. For the two point function one easily shows that

$$\begin{aligned}\langle 0|\ : e^{i\omega_1 X(\sigma_1)} :_N : e^{i\omega_2 X(\sigma_2)} :_N\ \|N\rangle\rangle &= \langle 0|\, e^{i(\omega_1+\omega_2)x_0}\, e^{i\omega_1(X_>(\sigma_1)-X_<(\sigma_1))} e^{2i\omega_2 X_<(\sigma_2)}\,\|N\rangle\rangle \\ &= \left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]^{\frac{\omega_1\omega_2}{2}}\ \langle 0|\, e^{i(\omega_1+\omega_2)x_0}\,\|N\rangle\rangle \\ &= \delta(\omega_1+\omega_2)\,\left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]^{-\frac{\omega_1^2}{2}}\,,\end{aligned} \tag{2.16}$$

where we have used the Baker-Campbell-Hausdorff (BCH) formula,

$$e^{B+C} = e^B\, e^{\frac{1}{2}[C,B]}\, e^C\,, \tag{2.17}$$

that holds provided that $[B,C]$ commutes with both B and C, as well as (2.4).

[2] We thank Andreas Recknagel for helping us understand this issue.

The boundary correlation functions for the boundary state with the boundary deformation (1.5) turned on, can be calculated by replacing the Neumann boundary state $\|N\rangle\rangle$ with (2.10). (As is also shown in the appendix, the above interpretation of this calculation as a limit of a bulk calculation is also valid in this context; in particular, our boundary vertex operators are therefore still normalised so that this bulk-boundary OPE coefficient is unity.) Thus the two-point function of such boundary vertex operators should be

$$\mathcal{A}_2 = \langle 0| \, : e^{i\omega_1 X(\sigma_1)} :_N : e^{i\omega_2 X(\sigma_2)} :_N \, \|B\rangle\rangle \, . \tag{2.18}$$

Performing the same calculation as above, one finds that $\mathcal{A}_2$ equals

$$\mathcal{A}_2 = \left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]^{\frac{\omega_1\omega_2}{2}} \langle 0| \, e^{i(\omega_1+\omega_2)x_0} \, e^{i\omega_1(X_>(\sigma_1)-X_<(\sigma_1))+i\omega_2(X_>(\sigma_2)-X_<(\sigma_2))} \, \|B\rangle\rangle \, . \tag{2.19}$$

The exponential on the right hand side of (2.19) can be written as

$$\exp\left[i\omega_1(X_>(\sigma_1)-X_<(\sigma_1))+i\omega_2(X_>(\sigma_2)-X_<(\sigma_2))\right] \equiv \exp(Y) \, , \tag{2.20}$$

where

$$Y = -\frac{1}{2}\sum_{n\neq 0}\frac{1}{|n|}\,(\alpha_n+\tilde{\alpha}_{-n})\gamma_n \, , \tag{2.21}$$

and γ_n is given as

$$\gamma_n = \omega_1\, e^{-in\sigma_1} + \omega_2\, e^{-in\sigma_2} \, . \tag{2.22}$$

Now we observe that

$$e^Y \|B\rangle\rangle = e^Y P_\infty \, e^{-\lambda J_0^+} \|e\rangle\rangle = P_\infty \, e^Y e^{-\lambda J_0^+} \|e\rangle\rangle = P_\infty \, e^Z \|e\rangle\rangle \, , \tag{2.23}$$

where we have used that P_∞ commutes with e^Y (since Y does not involve any zero modes), and where

$$Z = -\lambda \sum_{k=0}^{\infty}\frac{1}{k!}\,\mathrm{Ad}^k(Y) J_0^+ \, . \tag{2.24}$$

Here $\mathrm{Ad}^k(Y)J_0^+$ is the k-fold commutator $[Y,[Y,\cdots,[Y,Z]\cdots]]$ and we have used that $Y\|e\rangle\rangle = 0$ and that the commutator $[Y,J_0^+]$ is a sum of terms involving only the modes J_l^+. Using that $J_n^3 = \frac{1}{\sqrt{2}}\alpha_n$, as well as the SU(2) commutation relations, one finds that

$$Z = -\lambda\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{1}{\sqrt{2}}\right)^k \sum_{n_1,\ldots,n_k}\frac{\gamma_{n_1}}{|n_1|}\cdots\frac{\gamma_{n_k}}{|n_k|}\, J^+_{n_1+\cdots+n_k} \, , \tag{2.25}$$

where the sum runs over all $n_i \neq 0$. The total amplitude is then

$$\begin{aligned}\mathcal{A}_2 &= \left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]^{\frac{\omega_1\omega_2}{2}} \langle 0| \, e^{i(\omega_1+\omega_2)x_0} \, P_\infty \, e^Z \, \|e\rangle\rangle \\ &= \left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]^{\frac{\omega_1\omega_2}{2}} \langle 0| \, e^{i(\omega_1+\omega_2)x_0} \, e^Z \, \|e\rangle\rangle \, ,\end{aligned} \tag{2.26}$$

since the state on the left is in the space onto which P_∞ projects. For each k the coefficient of J^+_m in Z is proportional to

$$
\begin{aligned}
D_m &= \sum_{n_1,\dots,n_k} \frac{\gamma_{n_1}}{|n_1|}\cdots\frac{\gamma_{n_k}}{|n_k|}\,\delta_{n_1+\cdots+n_k,m} \\
&= \int_0^{2\pi}\frac{dt}{2\pi}\sum_{n_1,\dots,n_k} e^{-it(n_1+\cdots+n_k-m)}\,\frac{\gamma_{n_1}}{|n_1|}\cdots\frac{\gamma_{n_k}}{|n_k|} \\
&= \int_0^{2\pi}\frac{dt}{2\pi}\,e^{itm}\left(\sum_{n\neq 0}\frac{\gamma_n}{|n|}e^{-itn}\right)^k . \qquad (2.27)
\end{aligned}
$$

With γ_n given by (2.22) the above sum can be performed explicitly,

$$
\begin{aligned}
\sum_{n\neq 0}\frac{\gamma_n}{|n|}e^{-itn} &= -\omega_1\log\left(1-e^{-i(\sigma_1+t)}\right)-\omega_1\log\left(1-e^{i(\sigma_1+t)}\right)- \\
&\quad -\omega_2\log\left(1-e^{-i(\sigma_2+t)}\right)-\omega_2\log\left(1-e^{i(\sigma_2+t)}\right). \qquad (2.28)
\end{aligned}
$$

Thus the J^+_m component of Z becomes

$$
\begin{aligned}
Z_m &= -\lambda\,J^+_m\int_0^{2\pi}\frac{dt}{2\pi}\,e^{itm}\exp\left(-\frac{1}{\sqrt{2}}\sum_{n\neq 0}\frac{\gamma_n}{|n|}e^{-itn}\right) \\
&= -\lambda\,J^+_m\int_0^{2\pi}\frac{dt}{2\pi}\,e^{itm}\left(1-e^{-i(\sigma_1+t)}\right)^{\frac{\omega_1}{\sqrt{2}}}\left(1-e^{i(\sigma_1+t)}\right)^{\frac{\omega_1}{\sqrt{2}}}\times \\
&\quad\times\left(1-e^{-i(\sigma_2+t)}\right)^{\frac{\omega_2}{\sqrt{2}}}\left(1-e^{i(\sigma_2+t)}\right)^{\frac{\omega_2}{\sqrt{2}}} \\
&= -\lambda\,J^+_m\int_0^{2\pi}\frac{dt}{2\pi}\,e^{itm}\left(4\sin^2\left(\frac{\sigma_1+t}{2}\right)\right)^{\frac{\omega_1}{\sqrt{2}}}\left(4\sin^2\left(\frac{\sigma_2+t}{2}\right)\right)^{\frac{\omega_2}{\sqrt{2}}} . \qquad (2.29)
\end{aligned}
$$

Summing over all m we therefore obtain

$$
Z = -\lambda\int_0^{2\pi}\frac{dt}{2\pi}\,J^+(e^{-it})\left(4\sin^2\left(\frac{\sigma_1+t}{2}\right)\right)^{\frac{\omega_1}{\sqrt{2}}}\left(4\sin^2\left(\frac{\sigma_2+t}{2}\right)\right)^{\frac{\omega_2}{\sqrt{2}}} , \qquad (2.30)
$$

where

$$
J^+(e^{-it}) = \sum_{m\in\mathbb{Z}} J^+_m e^{itm} . \qquad (2.31)
$$

Finally we use that

$$
\langle 0|\,e^{i(\omega_1+\omega_2)x_0}\,J^+(e^{-it_1})\cdots J^+(e^{-it_m})\,\|e\rangle\!\rangle = \delta(\omega_1+\omega_2+\sqrt{2}m)\prod_{r<s}^{m}\left[4\sin^2\left(\frac{t_r-t_s}{2}\right)\right], \qquad (2.32)
$$

as follows from (2.7), performing a similar calculation as in (2.16). (The only minor difference is that now α_n with $n>0$ does not annihilate the boundary state $\|e\rangle\!\rangle$, but rather $\alpha_n\|e\rangle\!\rangle = -\tilde{\alpha}_{-n}\|e\rangle\!\rangle$. This right-moving creation mode then annihilates the in-vacuum to the left.)

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Putting everything together, we therefore find that the two-point function can be evaluated as

$$\begin{aligned}\mathcal{A}_2 = &\left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]^{\frac{\omega_1\omega_2}{2}} \sum_{m=0}^{\infty}\frac{(-\lambda)^m}{m!}\delta(\omega_1+\omega_2+\sqrt{2}m)\,\times \\ &\times \prod_{l=1}^{m}\int\frac{dt_l}{2\pi}\prod_{k=1}^{2}\left(4\sin^2\left(\frac{\sigma_k+t_l}{2}\right)\right)^{\frac{\omega_k}{\sqrt{2}}}\prod_{r<s}^{m}\left[4\sin^2\left(\frac{t_r-t_s}{2}\right)\right]. \end{aligned} \tag{2.33}$$

For a given ω_1 and ω_2, at most one term in the sum over m contributes.

The calculation of the two point function can easily be generalised to an n-point boundary correlator

$$\begin{aligned}\mathcal{A}_n &= \langle 0|\;:e^{i\omega_1 X(\sigma_1)}:_N\,:e^{i\omega_2 X(\sigma_2)}:_N\cdots:e^{i\omega_n X(\sigma_n)}:_N\;\|B\rangle\!\rangle \\ &= \prod_{i<j}\left[4\sin^2\left(\frac{\sigma_i-\sigma_j}{2}\right)\right]^{\frac{\omega_i\omega_j}{2}}\langle 0|\,e^{i(\omega_1+\omega_2+\cdots+\omega_n)x_0}\,e^{Z}\,\|e\rangle\!\rangle\,, \end{aligned}\tag{2.34}$$

where Z is now given by

$$Z = -\lambda\int_0^{2\pi}\frac{dt}{2\pi}\,J^+(e^{-it})\prod_{i=1}^{n}\left(4\sin^2\left(\frac{\sigma_i+t}{2}\right)\right)^{\frac{\omega_i}{\sqrt{2}}}. \tag{2.35}$$

The n-point function (2.34) can thus be evaluated as a perturbation series in λ,

$$\begin{aligned}\mathcal{A}_n = &\prod_{i<j}\left[4\sin^2\left(\frac{\sigma_i-\sigma_j}{2}\right)\right]^{\frac{\omega_i\omega_j}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda)^m}{m!}\prod_{l=1}^{m}\int\frac{dt_l}{2\pi}\prod_{k=1}^{n}\left(4\sin^2\left(\frac{\sigma_k+t_l}{2}\right)\right)^{\frac{\omega_k}{\sqrt{2}}}\times \\ &\times\langle 0|\,e^{i(\omega_1+\cdots+\omega_n)x_0}J^+(e^{-it_1})\cdots J^+(e^{-it_m})\,\|e\rangle\!\rangle\,. \end{aligned}\tag{2.36}$$

Using (2.32) we then obtain

$$\begin{aligned}\mathcal{A}_n = &\prod_{i<j}\left[4\sin^2\left(\frac{\sigma_i-\sigma_j}{2}\right)\right]^{\frac{\omega_i\omega_j}{2}}\sum_{m=0}^{\infty}\frac{(-\lambda)^m}{m!}\delta(\omega_1+\omega_2+\cdots+\omega_n+\sqrt{2}m)\,\times \\ &\times\prod_{l=1}^{m}\int\frac{dt_l}{2\pi}\prod_{k=1}^{n}\left(4\sin^2\left(\frac{\sigma_k+t_l}{2}\right)\right)^{\frac{\omega_k}{\sqrt{2}}}\prod_{r<s}^{m}\left(4\sin^2\left(\frac{t_r-t_s}{2}\right)\right). \end{aligned}\tag{2.37}$$

Again, for given ω_i, at most one term in the sum over m contributes. We recognise this formula as the perturbative open string expansion, where one calculates Neumann boundary correlators involving the n fields labelled by ω_i, as well as an arbitrary number of perturbing fields J^+. [Indeed, the correlation functions that appear in (2.37) are precisely the standard Neumann boundary Greens functions.] This formula was, for example, used as a starting point in [27].

The above amplitudes (2.33) and (2.37) are formally divergent. As is clear from the arguments of the appendix, they are to be understood as the limit of (A.22) (or its analogue for the case of the n-point amplitude) as $\tau_i \to 0$. This gives a prescription for how to evaluate them. (In the above we have assumed that this limit exists. If this is not the case, the resulting expressions need to be regularised further by subtracting off the contributions from open string operators with smaller conformal weight that appear when the bulk operators approach the boundary. Unfortunately we have not been able to understand the singularity structure of the correlators as $\tau_i \to 0$ in general.)

2.3 Cylinder partition function

For the euclidean theory the boundary state formalism can also be used to calculate the cylinder diagram and thus determine the open string spectrum. This is possible since in the boundary state of the euclidean theory

$$\|B\rangle\!\rangle = \sum_j \sum_{m\geq 0}^{j} \binom{j+m}{2m} (-\lambda)^{2m} \, |j,m,m\rangle\!\rangle \tag{2.38}$$

the Ishibashi states $|j,m,m\rangle\!\rangle$ are pairwise orthogonal. The calculation is most easily done using a trick that was first described in [9]. As was mentioned before, the boundary state $\|B\rangle\!\rangle$ is the projection of the usual $\mathrm{SL}(2,\mathbb{C})$ boundary state

$$\|g\rangle\!\rangle = \sum_{j,m,n} D^j_{-m,n}(g) \, |j,m,n\rangle\!\rangle \tag{2.39}$$

corresponding to the group element $g = g_\lambda$ (1.7), where one projects onto states for which $m = n$. In terms of the matrix elements of $\mathrm{SL}(2,\mathbb{C})$ this projection can be described as

$$\frac{1}{\pi}\int_0^\pi d\theta \, D^j_{-m,n}\left(\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} g_\lambda \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \right) . \tag{2.40}$$

Up to an overall normalisation the cylinder diagram between two boundary states with λ_1 and λ_2 is therefore

$$\frac{1}{\pi}\int_0^\pi d\theta \sum_{j\in\frac{1}{2}\mathbf{Z}_+} \mathrm{Tr}_j \left[\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} g_{\lambda_1}^{-1} \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} g_{\lambda_2} \right] \chi_{j^2}(q) \, , \tag{2.41}$$

where $\chi_{j^2}(q)$ is given by

$$\chi_{j^2}(q) = \vartheta_{\sqrt{2}j}(q) - \vartheta_{\sqrt{2}(j+1)}(q) \, , \tag{2.42}$$

with

$$\vartheta_s(q) = \frac{q^{\frac{1}{2}s^2}}{\eta(q)} \, . \tag{2.43}$$

For each θ, the trace in (2.41) is simply $\frac{\sin((2j+1)\alpha)}{\sin\alpha}$ for some $\alpha \equiv \alpha(\lambda_1,\lambda_2,\theta)$. In order to determine α, consider the $j = 1/2$ representation for which we get

$$\begin{aligned} 2\cos\alpha &= \mathrm{tr}\left[\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \begin{pmatrix} 1 & \lambda_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \begin{pmatrix} 1 & -\lambda_2 \\ 0 & 1 \end{pmatrix} \right] \\ &= 2\cos(2\theta) \, , \end{aligned} \tag{2.44}$$

from which it follows that $\alpha = 2\theta$. Thus the open string spectrum is

$$\frac{1}{\eta(\tilde{q})} \sum_{n\in\mathbb{Z}} \int_0^1 ds\, \tilde{q}^{(n+s)^2} = \frac{1}{\eta(\tilde{q})} \int_{-\infty}^{\infty} ds\, \tilde{q}^{s^2} . \tag{2.45}$$

In particular, it follows that (2.44) is independent of λ_1 and λ_2. Furthermore, the conformal weights of the open string states are real and non-negative (and not all equal to zero), corresponding to arbitrary real momenta in the open string. This is then nicely in accord with our calculation for the open string two-point function where the boundary vertex operators that are obtained as one takes the bulk operators to the boundary are also labelled by arbitrary real momenta.

3. Continuation to timelike signature

As we have mentioned in the introduction, the euclidean boundary conformal field theory we have discussed up to now is related by an analytic continuation $X \mapsto -iX_0$ to the timelike boundary conformal field theory describing the half S-brane. This Wick rotation is however rather subtle, and some of its aspects have not been satisfactorily understood. According to Sen's original proposal,[3] the Wick rotation should be performed for physically meaningful quantities in the corresponding string theory; this can be justified because this analytic continuation maps string solutions of the euclidean theory to string solutions of the timelike theory. However, it is less clear to what extent this analytic continuation can also be performed at the level of the underlying conformal field theory.

The prime example for which Sen's proposal can be applied concerns the 'profile' $f(x)$ of the D-brane. Physically the function $f(x)$ can be interpreted as determining the transverse components of the stress energy tensor $T_{ij} = -\mathcal{T}_p\, f(x)\, \delta_{ij}$ of the brane. As was shown in [2] it can be determined from the bulk 1-point functions, which for the above D-brane in the euclidean theory are given by

$$\tilde{f}(p) \equiv \langle 0|\, e^{-ipx}\, \|g_\lambda\rangle\rangle = (-\lambda)^{\frac{p}{\sqrt{2}}}\, \delta_{p,\sqrt{2}\mathbb{N}_0} . \tag{3.1}$$

The profile of the D-brane is then simply the Fourier transform of $\tilde{f}(p)$,

$$f(x) = \frac{1}{1+\lambda\, e^{-i\sqrt{2}x}} . \tag{3.2}$$

This function can be Wick-rotated, leading to

$$g(x) = \frac{1}{1+\lambda\, e^{\sqrt{2}x}} . \tag{3.3}$$

This then describes the profile of the timelike D-brane; its Fourier coefficients [10]

$$\tilde{g}(p) = -\frac{\pi i}{\sqrt{2}}\, e^{-\frac{i}{\sqrt{2}}p\log(\lambda)} \frac{1}{\sinh(\pi p/\sqrt{2})} \tag{3.4}$$

[3] We thank Ashoke Sen for a detailed discussion about this issue.

can then be identified with the bulk 1-point functions of the timelike theory. This procedure thus defines a method for the 'analytic continuation' of the bulk 1-point function (3.1) to (3.4). This example also shows that this analytic continuation may be quite subtle from the point of view of the underlying conformal field theory.

Actually, the knowledge of (3.4) appears to be sufficient to determine the boundary state in the timelike theory uniquely.[4] For a timelike boson, the states of a given real momentum $p \neq 0$ form already an irreducible Virasoro representation, and thus there is a Virasoro Ishibashi state $|p\rangle\rangle$ for each such momentum. (This Ishibashi state is of 'Dirichlet type', i.e. it is explicitly given by

$$|p\rangle\rangle = \exp\Big(- \sum_{n>0} \frac{1}{n} \alpha^0_{-n} \bar{\alpha}^0_{-n} \Big)\, |p\rangle\,, \tag{3.5}$$

where $|p\rangle$ is the momentum ground state with momentum p, and we have denoted the modes of the timelike boson by α^0_n and $\tilde{\alpha}^0_n$.) The only subtlety concerns the sector with $p = 0$, since there are infinitely many Virasoro Ishibashi states with $p = 0$ that are labelled by a positive integer. The bulk 1-point functions we have discussed above determine already uniquely the coefficients in front of the Ishibashi states corresponding to real momenta with $p \neq 0$; if we ignore the subtlety concerning the sector with $p = 0$, the boundary state must therefore be of the 'integrated form' [10, 28]

$$\| B\rangle\rangle^{(T)} = \int dp\, \frac{\lambda^{-ip/\sqrt{2}}}{\sinh(\pi p/\sqrt{2})}\, |p\rangle\rangle\,. \tag{3.6}$$

However, this argument misses non-renormalisable components of the boundary state, i.e. contributions from Ishibashi states that come from sectors where the momentum is not real. These non-renormalisable parts are probably important for the calculation of higher point functions [27], the consistency of the bulk-boundary OPE and the presence of infinitely many conserved charges in two dimensional string theory [19]. They may also be needed in order to make sense of the cylinder diagram of two such boundary states [29].

At any rate, it would be interesting to determine physical quantities of the timelike theory using the above analytic continuation from the euclidean description. This should be possible not just for the bulk one-point functions, but also for more complicated correlators. It would then be interesting to compare these results with what can be directly computed from the boundary state (3.6), using for example the techniques described in this paper.

Finally, we should mention that while the boundary deformation (1.5) in the euclidean theory is not hermitean, this does not mean that it is without any physical significance. Non-hermitean quantum mechanical systems and defect conformal field theories have played an important role in condensed matter theory (for a recent example see [30]). Thus the results for the correlation functions of the euclidean theory are also interesting in their own right.

[4] This was already pointed out in [28].

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A. Boundary correlators as limits of bulk correlators

The same methods which were used in section 2 to calculate boundary correlators are also useful to calculate correlation functions involving bulk operators. In the context of the rolling tachyon theory, such bulk correlators are related to corrections of the decay of the unstable D-brane into closed strings. In this appendix we focus on the relation between the bulk and the boundary vertex operators. In particular we want to show that the open string normal ordering (2.12) can be derived by taking the leading term as the bulk vertex operators approach the boundary. In the following we shall first discuss the Neumann case before applying the relevant ideas to the case with a non-trivial boundary deformation.

A.1 The simple Neumann case

It is instructive to consider first the simple case of a standard Neumann boundary condition. The two point function $\mathcal{B}_2^N$ of two bulk vertex operators is given by

$$\mathcal{B}_2^N = \langle 0| \, : e^{i\omega_1 X(\sigma_1,\tau_1)} :: e^{i\omega_2 X(\sigma_2,\tau_2)} : \, \|N\rangle\!\rangle \, , \tag{A.1}$$

where the normal ordering is now the standard closed string normal ordering prescription, i.e.

$$: e^{i\omega X(\sigma,\tau)} := e^{i\omega X_<(\sigma,\tau)} \, e^{i\omega x_0} \, e^{\frac{i}{2}\omega p \tau} \, e^{i\omega X_>(\sigma,\tau)} \, , \tag{A.2}$$

with

$$X_>(\sigma,\tau) = \frac{i}{2} \sum_{n>0} \frac{1}{n} e^{-in\tau} \left(\alpha_n e^{-in\sigma} + \tilde{\alpha}_n e^{in\sigma} \right) , \tag{A.3}$$

and similarly for $X_<(\sigma,\tau)$. Here τ is purely imaginary, and lies in fact in the lower half-plane. Since the negative modes annihilate the vacuum to the left, we can eliminate them by commuting them through to the left. Applying the BCH formula we then get

$$\begin{aligned} \mathcal{B}_2^N = & \left[4 e^{-i(\tau_1-\tau_2)} \sin\left(\frac{\sigma_1+\tau_1-(\sigma_2+\tau_2)}{2} \right) \sin\left(\frac{\sigma_1-\tau_1-(\sigma_2-\tau_2)}{2} \right) \right]^{\frac{\omega_1\omega_2}{4}} \times \\ & \times \langle 0| \, e^{i\omega_1 x_0} \, e^{\frac{i}{2}\omega_1 p \tau_1} \, e^{i\omega_2 x_0} \, e^{\frac{i}{2}\omega_2 p\tau_2} \, e^{i\omega_1 X_>(\sigma_1,\tau_1)} \, e^{i\omega_2 X_>(\sigma_2,\tau_2)} \, \|N\rangle\!\rangle \, , \end{aligned} \tag{A.4}$$

where we have used that

$$\begin{aligned} [X_>(\sigma_1,\tau_1), X_<(\sigma_2,\tau_2)] = & -\frac{1}{4} \log\Big[4 e^{-i(\tau_1-\tau_2)} \sin\left(\frac{\sigma_1+\tau_1-(\sigma_2+\tau_2)}{2} \right) \times \\ & \times \sin\left(\frac{\sigma_1-\tau_1-(\sigma_2-\tau_2)}{2} \right) \Big] \, . \end{aligned} \tag{A.5}$$

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Next we move the p-mode to the right, using that $p\,\|N\rangle\rangle = 0$, and obtain

$$\begin{aligned}\mathcal{B}_2^N &= e^{\frac{i}{2}\omega_1\,\omega_2\tau_1}\left[4e^{-i(\tau_1-\tau_2)}\sin\left(\frac{\sigma_1+\tau_1-(\sigma_2+\tau_2)}{2}\right)\sin\left(\frac{\sigma_1-\tau_1-(\sigma_2-\tau_2)}{2}\right)\right]^{\frac{\omega_1\omega_2}{4}}\times\\ &\quad\times\langle 0|\,e^{i(\omega_1+\omega_2)x_0}\,e^{i\omega_1 X_>(\sigma_1,\tau_1)+i\omega_2 X_>(\sigma_2,\tau_2)}\,\|N\rangle\rangle\,.\end{aligned}\tag{A.6}$$

It remains to simplify the exponential of the positive modes. To this end we write

$$\begin{aligned}e^{i\omega_1 X_>(\sigma_1,\tau_1)+i\omega_2 X_>(\sigma_2,\tau_2)} &= e^{i\omega_1 X_<(\sigma_1,-\tau_1)+i\omega_2 X_<(\sigma_2,-\tau_2)}\,e^{C}\times\\ &\quad\times e^{i\omega_1(X_>(\sigma_1,\tau_1)-X_<(\sigma_1,-\tau_1))+i\omega_2(X_>(\sigma_2,\tau_2)-X_<(\sigma_2,-\tau_2))}\,,\end{aligned}\tag{A.7}$$

where C is the commutator term (as follows from the BCH formula),

$$C=\frac{1}{2}\left[i\omega_1 X_>(\sigma_1,\tau_1)+i\omega_2 X_>(\sigma_2,\tau_2),i\omega_1 X_<(\sigma_1,-\tau_1)+i\omega_2 X_<(\sigma_2,-\tau_2)\right].\tag{A.8}$$

The idea of this construction is that the exponential that appears to the right in (A.7) annihilates the Neumann boundary state, whereas the exponential to the left annihilates the vacuum. Thus it only remains to determine C, which using (A.5) equals

$$\begin{aligned}C &= \frac{\omega_1^2}{8}\log\left[-4e^{-2i\tau_1}\sin^2(\tau_1)\right]+\frac{\omega_2^2}{8}\log\left[-4e^{-2i\tau_2}\sin^2(\tau_2)\right]+\\ &\quad+\frac{\omega_1\omega_2}{8}\log\left[4e^{-i(\tau_1+\tau_2)}\sin\left(\frac{\tau_1+\tau_2+\sigma_2-\sigma_1}{2}\right)\sin\left(\frac{-(\tau_1+\tau_2)+\sigma_2-\sigma_1}{2}\right)\right]+\\ &\quad+\frac{\omega_1\omega_2}{8}\log\left[4e^{-i(\tau_1+\tau_2)}\sin\left(\frac{\tau_1+\tau_2+\sigma_1-\sigma_2}{2}\right)\sin\left(\frac{-(\tau_1+\tau_2)+\sigma_1-\sigma_2}{2}\right)\right].\end{aligned}\tag{A.9}$$

The total amplitude is then equal to zero unless $\omega_1+\omega_2=0$; if this is the case, it equals

$$\begin{aligned}\mathcal{B}_2^N &= e^{-\frac{i}{2}\omega^2\tau_1}\left[4e^{-i(\tau_1-\tau_2)}\sin\left(\frac{\sigma_1+\tau_1-(\sigma_2+\tau_2)}{2}\right)\sin\left(\frac{\sigma_1-\tau_1-(\sigma_2-\tau_2)}{2}\right)\right]^{-\frac{\omega^2}{4}}\times\\ &\quad\times\left[-4e^{-2i\tau_1}\sin^2(\tau_1)\right]^{\frac{\omega^2}{8}}\left[-4e^{-2i\tau_2}\sin^2(\tau_2)\right]^{\frac{\omega^2}{8}}\times\\ &\quad\times\left[4e^{-i(\tau_1+\tau_2)}\sin\left(\frac{\tau_1+\tau_2+\sigma_2-\sigma_1}{2}\right)\sin\left(\frac{-(\tau_1+\tau_2)+\sigma_2-\sigma_1}{2}\right)\right]^{-\frac{\omega^2}{8}}\times\\ &\quad\times\left[4e^{-i(\tau_1+\tau_2)}\sin\left(\frac{\tau_1+\tau_2+\sigma_1-\sigma_2}{2}\right)\sin\left(\frac{-(\tau_1+\tau_2)+\sigma_1-\sigma_2}{2}\right)\right]^{-\frac{\omega^2}{8}},\end{aligned}\tag{A.10}$$

where $\omega=\omega_1=-\omega_2$, say. The bulk vertex operators approach the boundary when $\tau_1,\tau_2\to 0$. In this limit the amplitude goes to zero as

$$\sin(\tau_1)^{\frac{\omega_1^2}{4}}\,\sin(\tau_2)^{\frac{\omega_2^2}{4}}\,.\tag{A.11}$$

If we divide by this term in order to obtain the leading behavior in this limit, we obtain

$$\mathcal{B}_2^N\simeq\left[4\sin^2\left(\frac{\sigma_1-\sigma_2}{2}\right)\right]^{-\frac{\omega^2}{2}},\tag{A.12}$$

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which therefore does indeed agree with (2.16). Furthermore, using the general structure of the bulk-boundary OPE we can deduce the conformal weight of the boundary field. Indeed, the exponent of (A.12) can be identified with

$$-\frac{\omega^2}{4} = h_\omega + \bar{h}_\omega - h_\omega^{op}\,, \tag{A.13}$$

where h_ω and $\bar{h}_\omega$ are the left- and right-moving conformal weights of the bulk operator, while h_ω^{op} is the conformal weight of the corresponding boundary operator. With our conventions,

$$h_\omega = \bar{h}_\omega = \frac{\omega^2}{8}\,, \tag{A.14}$$

thus leading to

$$h_\omega^{op} = \frac{\omega^2}{2}\,, \tag{A.15}$$

which is indeed in agreement with the conformal behavior of (A.12).

A.2 Non-trivial boundary condensate

The calculation of the bulk two point function $\mathcal{B}_2$, where we replace the Neumann boundary state $\|N\rangle\rangle$ by $\|B\rangle\rangle$, is almost identical. The only difference is that now the operator $\hat{Y}$ in

$$\exp(\hat{Y}) = \exp\left(i\omega_1(X_>(\sigma_1,\tau_1) - X_<(\sigma_1,-\tau_1)) + i\omega_2(X_>(\sigma_2,\tau_2) - X_<(\sigma_2,-\tau_2))\right) \tag{A.16}$$

does not annihilate $\|B\rangle\rangle$ any more. However, it still has an expansion of the form

$$\hat{Y} = -\frac{1}{2}\sum_{n\neq 0}\frac{1}{|n|}\,(\alpha_n + \tilde{\alpha}_{-n})\,\hat{\gamma}_n\,, \tag{A.17}$$

where $\hat{\gamma}_n$ is now

$$\hat{\gamma}_n = \omega_1 e^{-i|n|\tau_1}e^{-in\sigma_1} + \omega_2 e^{-i|n|\tau_2}e^{-in\sigma_2}\,. \tag{A.18}$$

In particular, we can use essentially the same calculation as in (2.20)–(2.25), the only minor difference being that now

$$\begin{aligned}\sum_{n\neq 0}\frac{\hat{\gamma}_n}{|n|}e^{-itn} &= -\omega_1\log\left(1-e^{-i(\sigma_1+\tau_1+t)}\right) - \omega_1\log\left(1-e^{i(\sigma_1-\tau_1+t)}\right) - \\ &\quad -\omega_2\log\left(1-e^{-i(\sigma_2+\tau_2+t)}\right) - \omega_2\log\left(1-e^{i(\sigma_2-\tau_2+t)}\right).\end{aligned} \tag{A.19}$$

The relevant Z is therefore now

$$\begin{aligned}Z &= -\lambda\int_0^{2\pi}\frac{dt}{2\pi}\,J^+(e^{-it})\exp\left(-\frac{1}{\sqrt{2}}\sum_{n\neq 0}\frac{\hat{\gamma}_n}{|n|}e^{-itn}\right)\\ &= -\lambda\int_0^{2\pi}\frac{dt}{2\pi}\,J^+(e^{-it})\left(1-e^{-i(\sigma_1+\tau_1+t)}\right)^{\frac{\omega_1}{\sqrt{2}}}\left(1-e^{i(\sigma_1-\tau_1+t)}\right)^{\frac{\omega_1}{\sqrt{2}}}\times\\ &\quad\times\left(1-e^{-i(\sigma_2+\tau_2+t)}\right)^{\frac{\omega_2}{\sqrt{2}}}\left(1-e^{i(\sigma_2-\tau_2+t)}\right)^{\frac{\omega_2}{\sqrt{2}}}\,.\end{aligned} \tag{A.20}$$

Thus the total amplitude can be expressed as

$$\begin{aligned}\mathcal{B}_2 &= \langle 0| :e^{i\omega_1 X(\sigma_1,\tau_1)}::e^{i\omega_2 X(\sigma_2,\tau_2)}: \|B\rangle\rangle \\ &= \mathcal{B}_2^N \, \langle 0|e^{i(\omega_1+\omega_2)x_0} e^{Z}|e\rangle\rangle \,, \end{aligned} \tag{A.21}$$

where $\mathcal{B}_2^N$ is the two point function with Neumann boundary condition (A.10). Expansion as a power series in λ then gives

$$\begin{aligned}\mathcal{B}_2 = \mathcal{B}_2^N &\sum_{m=0}^{\infty} \frac{(-\lambda)^m}{m!} \delta(\omega_1+\omega_2+\sqrt{2}m) \prod_{i=1}^{m} \int_0^{2\pi} \frac{dt_i}{2\pi} \prod_{k<l}^{m} \left(4\sin^2\left(\frac{t_k-t_l}{2}\right)\right) \times \\ &\times \prod_{l=1}^{m} \Bigg[\left(1-e^{-i(\sigma_1+\tau_1+t_l)}\right)^{\frac{\omega_1}{\sqrt{2}}} \left(1-e^{i(\sigma_1-\tau_1+t_l)}\right)^{\frac{\omega_1}{\sqrt{2}}} \times \\ &\qquad \times \left(1-e^{-i(\sigma_2+\tau_2+t_l)}\right)^{\frac{\omega_2}{\sqrt{2}}} \left(1-e^{i(\sigma_2-\tau_2+t_l)}\right)^{\frac{\omega_2}{\sqrt{2}}} \Bigg]. \end{aligned} \tag{A.22}$$

Since the τ_i are purely imaginary (and lie in the lower half-plane), these integrals converge. In the limit $\tau_1, \tau_2 \to 0$ the factor $\mathcal{B}_2^N$ tends again to zero as in (A.11); if the integrals in (A.22) converge to a finite answer in this limit,[5] $\mathcal{B}_2$ goes to zero as

$$\sin(\tau_1)^{\frac{\omega_1^2}{4}} \, \sin(\tau_2)^{\frac{\omega_2^2}{4}} \,. \tag{A.23}$$

This implies, by the same reasoning as before, that we need to divide through by this factor, in order to isolate the leading contribution. The relevant limit then gives the 2-point boundary correlator of two non-trivial vertex operators with conformal weight $h_\omega^{op} = \omega^2/2$. On the other hand, it is clear by construction that in this limit $\mathcal{B}_2$ becomes the function $\mathcal{A}_2$ we have calculated in section 2.

It should be clear how this calculation can be generalised to the case of an n-point correlator, and that a limit analogous to (A.23) then leads to the function $\mathcal{A}_n$ we have calculated in section 2. Indeed, the main steps of this calculation are essentially identical to those in section 2.

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